Mental Models and the Meaning of Connectives:
A Study on Children, Adolescents and Adults

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Abstract

We present a study on the ability to comprehend conjunction, exclusive disjunction, bi-conditional and conditional. Mental model theory predicts differences in difficulty in dealing with such connectives, and it also predicts that it is easier to envisage situations that comply with an assertion than situations which do not comply. We carried out an experiment on children, adolescents and adults to validate these predictions within a developmental perspective. Participants had to judge some states of affairs as complying or not complying with sentences involving connectives. A further aim of the experiment was to test the power of the theory to account for connectives' comprehension within a pragmatic context. Thus, while half of the participants dealt with an abstract version of the task, the other half coped with a pragmatic version where the sentences were uttered by a character known as sincere in the complying condition and as a liar in the not complying condition. The results of the experiment show that difficulty in comprehension of the different connectives depends on the number of models they require. Also, the results show that it is easier to envisage situations complying with the meaning of a connective than situations which do not comply. The results hold for all groups of participants in both versions of the task. We conclude that mental model theory offers a plausible account of connectives' comprehension, which holds also within the investigated pragmatic context.

1. Introduction

Experimental data show that connectives vary in difficulty of comprehension. Conjunction *and* is handled by 2 years old children (Bloom, Lahey, Hood, Lifter & Fiess, 1980); disjunction *or* is understood after 4 years (Johansson & Sjolin, 1975); bi-conditional *only if then* emerges from 8 years of age but it is not fully mastered until 11;6 years (Staudenmayer & Bourne, 1977); conditional *if then* remains difficult even for 14 year old children, although around 5-6 years there is a clear improvement (Amidon, 1976; Staudenmayer & Bourne, 1977).

The connective interpretation varies from children to adolescents to adults. Conjunction is early understood as implying the co-occurrence of its constituents, even if children sometimes treat it as a disjunction (Johnson-Laird & Barres, 1994). Disjunction is commonly interpreted exclusively, namely as if it would imply a choice between the co-ordinated members and they should not be taken in combination; this kind of interpretation seems the favourite at every age (Staudenmayer & Bourne, 1977). To comprehend conditional relations, individuals have to grasp the possibility of relations between properties that are absent, but implied. The first achievement of this kind is the bi-conditional interpretation. Conditional interpretation is hardly caught even by adolescents and adults, who often interpret it as it would imply its converse, i.e. they usually give a bi-conditional interpretation (Taplin, 1971).

The meaning of the connectives has been mainly a concern of the theories on propositional reasoning, viz. the ability to reason with propositions and connectives. Some of these theories claim that the meaning of the connectives is conveyed by the truth-values they receive in a truth table system. For instance, Piaget and Inhelder (Inhelder & Piaget, 1958; Piaget, 1953) claim that people can construct true and false contingencies of propositions because they posses a mental logic: in their view, some truth functions and a set of transformations would develop by the early teens, so that children would grasp the meaning of the connectives.

Other theorists have proposed that the meaning of the connectives is grasped through natural deduction systems, where rules are claimed to have more psychological plausibility than standard logic (Braine, 1978; 1990; 1998, Braine & Rumain, 1983, Rips, 1990). In their view, the evaluation of contingencies complying with propositions would depend on the internal structure of the proposition itself: each connective would define which inferential rules can be applied and, therefore, how reasoners can envisage the correct contingencies.

In a radically alternative view, Pollard (1981) and Griggs and Cox (1982) claim that the understanding of the meaning of connectives depends on the reasoners' previous experience. In particular, the specific experiences encoded in memory would provide a set of domain dependent rules that reasoners can use in the current situation by analogy.

Cheng, Holyoak and colleagues analyze just the conditional connective and postulate the existence of abstract knowledge structures such as causation, obligation and permission (Cheng & Holyoak, 1985; Cheng, Holyoak, Nisbett & Oliver, 1986). They argue that the meaning of a conditional emerges from the concrete context within which it is introduced. For instance, a permission context would induce the reasoner to think about the possibility in which an action is done provided the precondition is satisfied.

All of the mentioned theories offer an explanation of how people represent the meaning of the connectives, but none of them gives a systematical account of their difference in terms of difficulty of comprehension.

Mental model theory offers such an account. Our investigation into the mental representations of the meaning of the connectives follows the tenets of this theory.

2. Mental Models for Connectives

Mental model theory (Johnson-Laird, 1983; Johnson-Laird & Byrne, 1991) claims that the meaning of the connectives is conveyed by mental models. A model is an analogical representation: it does reproduce the structure of the states of affairs perceived or described. For example, the theory claims that the meaning of an indicative conditional such as:

If you eat too many cakes, then you put on weight

is represented by the model representation:

[too many cakes]   put on weight
                     ...

Each row in the representation denotes a model of a separate possibility. The first model is explicit and satisfies the antecedent and the consequent, and the second model (dots) is implicit: it allows for the case in which the antecedent is not satisfied.

The construction of models is guided by the Principle of truth and the attempt to maintain as much information implicit as possible. According to the Principle of truth, each model represents only what is true in a particular possibility. Hence, the first model represents the possibility in which the antecedent (and then the consequent) is true. Reasoners do not represent the possibilities in which the antecedent is false. The theory postulates that reasoners make "mental footnotes" to keep track of this information, but that these footnotes are soon likely to be forgotten. To indicate these footnotes we use the square brackets and the dots. The square brackets indicate that the antecedent (i.e. too many cakes) has been exhaustively represented in relation to the occurrence of the consequent (i.e. put on weight), i.e. it can not occur in any other model of the conditional (see Johnson-Laird, Byrne & Schaeken, 1992). The dots denote the wholly implicit models, in which the antecedent is false. Therefore, the fully explicit representation of a conditional calls for three models. In our example:

too many cakes	put on weight
not- too many cakes	put on weight
not- too many cakes	not- put on weight

In essence, mental model theory (MMT hereafter) assumes that human reasoners tend not to represent information explicitly. In fact, the more information that has to be represented explicitly, the greater the load on the working memory, and so the initial models of a proposition represent as much information implicit as possible. Implicit information is not immediately available for processing, but it becomes available when it is made explicit (see e.g. Bara, Bucciarelli, Johnson-Laird & Lombardo, 1994).

The fully explicit models required by each connective and the implicit models which people tend to construct are in Table 1. In fact, constraints of working memory prevents people to imagine all the possible models of a connective, and because truth appears to be highly salient, people represent first what is true.

Table1: Mental models representing the meaning of connectives (fully explicit models).

Connectives	True instances		False instances	
	Implicit models	Explicit models		Explicit models
p *and* q	p q	p q		p ¬q ¬p q ¬p ¬q
p *or else* q	p q	p ¬q ¬p q		p q ¬p ¬q
only if p *then* q	p q …	p q ¬p ¬q		p ¬q ¬p q
if p *then* q	p q …	p q ¬p q ¬p ¬q		p ¬q

The models representing the false instances of a proposition (see Table 1) would be fleshed out afterwards, only if they are needed to make the deduction. Thus, the theory predicts that the mental representation of cases complying with the meaning of a connective (i.e. the true instances) is easier than that of not complying cases (i.e. the false instances). The prediction is confirmed by Barres and Johnson-Laird (1997). They carried out a study where the participants were asked to list the true and the false instances given an assertion, and they found that representing the false instances is more difficult than representing the true ones. Thus, they claim that there is no a direct way to imagine what is false and errors are likely to occur when listing the false instances. For example, given the assertion "A or B, or both", most of their subjects perform correctly, and list the following instances in which the assertion is true:

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       A
           B
       A    B
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Then, in order to infer the false instances, most subjects negate the true ones and list what follows:

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       not A
                   not B
       not A        not B
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while we know the only false instance is:

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       not A        not B
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The aim of our experiment is to validate the following predictions within a developmental perspective. First, the difficulty of comprehension of the different connectives depends on the number of models they require. Second, in line with the study by Barres and Johnson-Laird (1997), to envisage the false instances of a connective is harder than to envisage the true instances. Also, we derive a corollary prediction from MMT and from the fact that working memory abilities, such as encoding abilities and the time information which can be maintained in memory (Cowan, 1997; Towse, Hitch & Hutton, 1998), are good predictors of the performance of subjects belonging to different age groups. Thus, the ability to deal with the not complying conditions should increase with age; such ability requires keeping in memory the true instances of a connective while deriving the false ones.

We tested these predictions using two different protocols: one in which the connectives are presented within an abstract context and the other in which they occur in a pragmatic context, where they are uttered by a character describing a certain state of affairs. We expect the evaluation of instances complying with the utterance proffered by a sincere character to be easier than the evaluation of instances not complying with the utterance proffered by a liar. This prediction parallels the prediction concerning complying versus not complying conditions in the abstract version of the task. In particular, granted that 7 year olds do possess the ability to think in term of lies, the requested abilities in the two contexts might be the same. Thus, we should detect an analogy between evaluating true instances of a sentence and understanding a person who is telling the truth, and between evaluating false instances of a sentence and understanding a person who is lying. In previous studies, MMT has been proved to account for the ability to comprehend connectives within different contexts, calling for the same basic principles (see, e.g. Bara, Bucciarelli & Lombardo, 2001). As people in everyday life have to deal with *utterances* rather than with abstract *sentences*, MMT for the meaning of connectives has to hold within pragmatic contexts as well as in abstract contexts.

To sum up, our aim is a validation of the following predictions:

i. The difficulty of comprehension of the meaning of the connectives depends on the number of models they require.

Thus, we expect to observe the following trend of difficulty, from the easiest to the most difficult connective: conjunction (one model), exclusive disjunction and bi-conditional (two models), conditional (three models).

ii. The evaluation of cases not complying with a connective is more difficult than the evaluation of complying cases (from the Principle of truth). It requires first to represent the states of affairs consistent with the connective, then to negate them. Also, the ability to evaluate instances not complying with a connective would improve with age.

We expect these predictions to hold both in the abstract and in the pragmatic version of the task.

3. Experiment

Method

Participants. We tested a sample of 180 subjects, 60 in each of the following age groups: children from 7 to 7;9 years old, adolescents from 14 to 14;9 years old, adults from 21 to 24;9 years old. They were students from primary schools, high-schools and university residences, who took part in the experiment voluntarily. There was a balanced proportion of males and females in each group of participants.

Design. We devised two protocols: the Abstract Protocol and the Pragmatic Protocol. In the Abstract Protocol propositions were presented within an arbitrary context, whereas in the Pragmatic Protocol the context was provided by a character proffering the utterance. The participants were randomly assigned to one of the two protocols. Thus, half of the participants of each age group dealt with the Abstract Protocol and half with the Pragmatic Protocol.

In either protocol, subjects had to deal with two conditions: «complying» and «not complying». As for the Abstract Protocol, propositions in the complying condition were said to be true, while propositions in the not complying condition were said to be false. As for the Pragmatic Protocol, propositions in the complying condition were uttered by a character said to be sincere, while propositions in the not complying condition were uttered by a character said to be a liar. Each subject dealt with 8 propositions, 4 in the complying condition and 4 in the not complying condition. The order of presentation of propositions within each condition was determined at random.

In either protocol, after reading the proposition, the experimenter showed the subjects a set of 4 cards, each representing a possible state of affairs. For each card the experimenter asked the subject if it satisfied the proposition or not. The cards were presented in a random order.

Materials. In the Abstract Protocol we used the following materials:

- 8 sheets of paper; on each of them it was written a proposition containing one of the following connectives: *and*, *or*, *only if-then*, *if-then*. Each connective occurred in two propositions, but with different content;
- 8 series of cards: each series consisted of 4 cards. Given a proposition «A *connective* B» (for example, «There are an aeroplane *and* a car»), the four cards represented A and B together (aeroplane and car), A alone (aeroplane), B alone (car), and CD, two things different from the ones mentioned in the proposition (for example, train and boat). Four series of cards were used in the complying condition and four series in the not complying condition.

In the Pragmatic Protocol we used the following materials:
- the puppets Minnie and Lucy;
- 8 sheets of paper; on each of them it was written an utterance proffered by a character. Each utterance contained one of the following connectives: *and*, *or*, *only if-then*, *if-then*. Each connective occurred in two utterances, but with different content;
- 8 series of cards: each series consisted of 4 cards, as in the Abstract Protocol. Four series of cards were used in the complying condition and four series in the not complying condition.

Procedure. The participants were tested individually in a quiet room.

Abstract Protocol: complying condition. The participant was told that he will be presented with some true propositions. Then the experimenter showed the sheet of paper with the first proposition and read it. For example:

> «Either there is a parrot, or there is a fish, but not both». I'll show you some cards: you have to choose those satisfying the proposition.

Then, the experimenter showed one of the four cards (for example, the card representing the fish) and asked:

> Does a fish alone[1] satisfy the proposition?

and waited until the participant has judged the card as satisfying or not the proposition. Then, the experimenter showed another card of the set, questioned the participant and so on with the other cards.

The same procedure was followed with the other three propositions of the complying condition.

Abstract Protocol: not complying condition. The participant was told that he will be presented with some false propositions. The procedure was the same as that in the Abstract Protocol, complying condition.

Pragmatic Protocol: complying condition. Participant was introduced to a character, Minnie, said to be sincere and they are told that Minnie will have proffered utterances

about some cards she owns. The experimenter began with the first proposition. For example:

> Minnie says «On each of my cards, only if there is a candle, then there is a book». Remember that Minnie always says the truth. I'll show you some cards, and you have to tell me which cards belong to Minnie.

Then, the experimenter showed one of the four cards (for example, the card representing the book) and asked:

> Can a book alone belong to Minnie?

and waited until the participant has judged the card as belong or not to the sincere character. Then, the experimenter showed another card of the set, questioned the participant and so on with the other cards.

The same procedure was followed with the other three propositions of the complying condition.

Pragmatic Protocol: not complying condition. Participant was introduced to a character, Lucy, said to be a liar and they are told that Lucia will have proffered utterances about some cards she owns. The procedure was the same as that in the Pragmatic Protocol, complying condition.

4. Results

The score was computed assigning one mark for each correct response (the choice of a card which would have to be chosen and the non-choice of a card which would have not to be chosen). So, the maximum score participants could obtain in each trial was 4.

i. Trend of difficulty in comprehension of the different connectives.

The trend in difficulty of comprehension of the different connectives is confirmed in the Abstract Protocol (see Table 2).

Table 2: Mean scores obtained by participants in the Abstract Protocol.

Age groups	Connectives			Mean score
	and	*or/only if-then*	*if-then*	
7-7;9	3.28	2.85	1.97	2.70
14-14;9	3.58	3.26	1.88	2.91
21-24;9	3.87	3.33	2.10	3.10
Overall	3.58	3.15	1.98	2.90

Participants find it easier to comprehend the meaning of «*and*» than the meaning of «*or/only if-then*»: the difference is statistically significant in each age group (paired T Test; t value ranging from 2.607 to 9.406, p value ranging from <.001 to p=.007). Also, participants find it easier to comprehend the meaning of the connectives «*or/only if-then*» than the meaning of «*if-then*»: again, the difference is statistically significant in each age group (paired T test; t value ranging from 6.520 to 19.746, p value is always <.001).

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[1] The cards are intended to correspond to the instances pq, p not q, not p q and not p not q. Thus, as the participant received pq, p, q and rs, we wanted to clarify that an implicit negation is intended.

Also, the results show that the knowledge of the meaning of the different connectives does increase with age (ANOVA one-way; F= 7.593, p<.001).

The same results hold in the Pragmatic Protocol (see Table 3).

Table 3: Mean scores obtained by participants in the Pragmatic Protocol.

Age groups	Connectives			Mean score
	and	*or/only if-then*	*if-then*	
7-7;9	3.12	2.63	2.02	2.59
14-14;9	3.55	3.18	2.12	2.95
21-24;9	3.37	3.22	2.20	2.93
Overall	3.35	3.01	2.11	2.82

Participants find it easier to deal with «*and*» than with «*or/only if-then*», and the difference is statistically significant in each age group (paired T Test; t value ranging from 3.447 to 3.768, p value ranging from <.001 to p=.005). An exception are adults: their performance with the different connectives is in the predicted direction, however, the difference is not statistically significant (paired T Test; t= 1.260, p=.109). Further, the participants find it easier to deal with the connectives «*or/only if-then*» than with «*if-then*», and the difference is statistically significant in each age group (paired T Test; t value ranging from 4.389 to 12.853, p value is always <.001).

The results show that also within the pragmatic context the knowledge of the meaning of the different connectives does increase with age (ANOVA one-way; F= 7.142, p<.001).

ii. The evaluation of cases complying with a connective is easier than the evaluation of cases not complying.

In the Abstract Protocol the prediction is confirmed (see Table 4).

Table 4: Mean scores obtained by participants in the two conditions of the Abstract Protocol.

Age groups	*Complying*	*Not complying*
7-7;9	2.75	2.53
14-14;9	3.07	2.93
21-24;9	3.24	3.07
Overall	3.02	2.84

All groups of participants performed better in the complying condition than in the not complying condition: the difference is statistically significant in each age group (paired T Test; t value ranging from 1.772 to 3.553, p value ranging from <.002 to <.05).
Also, within the Abstract Protocol the ability to evaluate instances not complying with a connective improve with age, as we predicted (ANOVA one-way; F= 7.249, p<.001).

The same results hold in the Pragmatic Protocol (see Table 5).

Table 5: Mean scores obtained by participants in the two conditions of the Pragmatic Protocol.

Age groups	*Complying*	*Not complying*
7-7;9	2.79	2.40
14-14;9	3.21	2.81
21-24;9	3.08	2.92
Overall	3.03	2.71

All groups of participants performed better in the complying condition than in the not complying condition, and the difference is statistically significant in each age group (paired T Test; t value ranging from 1.746 to 3.972, p value ranging from <.001 to <.05). Also within the Pragmatic Protocol the ability to evaluate instances not complying with a connective improve with age, as we predicted (ANOVA one-way; F= 5.994, p<.004).

Thus, MMT predictions hold both within the Abstract and the Pragmatic Protocol.

5. Conclusions

The aim of the experiment was to test the power of MMT in explaining how people represent the meaning of connectives in their mind. The results of the experiment confirm our predictions.

First, the difficulty in comprehending the meaning of a connective depends on the number of mental models it requires. Our results show the following trend of difficulty among connectives in both the Abstract and the Pragmatic Protocol: conjunction is easier than disjunction and bi-conditional, and the latter are easier than conditional.

Second, MMT predicts that, according to the Principle of truth, evaluating instances of compliance is easier than evaluating instances of non-compliance. Our data confirm such a prediction in both contexts. We have argued, in line with MMT, that the evaluation of instances not consistent with a connective leads subjects to err because they have to imagine first the true instances and then to infer the false ones. The corollary prediction that the ability to evaluate instances of non-compliance does increase with the age is also confirmed.

Our results are consistent with the results obtained by Bucciarelli e Johnson-Laird (2001). They investigate reasoning with conditionals within contexts where subjects have to construct instances complying with an assertion (i.e. instances of truth and obedience) and instances not complying with an assertion (i.e. lie and disobedience). Their results show that, while in a selection task the not complying context improves the performance, in a comprehension task participants are better at constructing cases of compliance than cases of non-compliance.

Our results corroborate MMT's predictions in three different age groups (children, adolescents and adults). These results strengthen the theory, which is powerful enough to predict and explain the development of connectives' comprehension. In particular, MMT explains the different difficulty of connectives, the difference in difficulty of comprehension of the same connective (easier in complying conditions, and more difficult in not complying conditions) and, finally, how people represent the meaning of the connectives both within an abstract context and a pragmatic context.

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